MOBILITY OF ATMOSPHERIC VORTICES

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ABSTRACT. A rectilinear vortex, as an entity, has no tendency to be mobile in an unlimited mass of fluid. A single vortex in an atmospheric street would remain stationary but for the velocities caused by the other vortices in the street and also those caused by external forces as obtain in the directive field (Sen, 1943).

While discussing the mobility of vortices, it is necessary to visualise separately the groupings of the vortices according to sign and the dynamical state of the medium in which they move. The chief topic of this note is the travel of a tropical cyclone, although incidental references have been made to the travel of an anticyclone in Indian latitudes. As regards the thermodynamical state, it may be mentioned that recent observations of temperature over India suggest that the regions of double vortices are characterised by feeble horizontal gradients of potential temperature. A graphical method of forecasting the path of a tropical cyclone has been developed.

In treatises on aerodynamics, it is customary to idealise the vortices in a street into point or rectilinear vortices. The vertical core-cylinders, as depicited in the annexed diagram (normal vortex streets for December on maps of India and adjoining countries shown at two levels, 'generative' and 'directive,' in perspective), are lines of concentrated vorticity or rectilinear vortices (Lamb, 1930), with small circular section. For practical purposes, the circular cross-section may be taken to represent a solid rotation in the presence of neutral air U_0 (irrotational and stagnant) so that the rectilinear vortices in the street behave as truly solid rotating cylinders with circulation round them.

The wind gradient, inside the two walls (Glauert, 1926) of dilatation and compression (outflow and inflow in a wake) increases, remains constant or decreases towards the neutral point, according as the angle between the walls is greater than, equal to or less than, a right angle. The isopleths of wind speed aloft in the daily charts confirm this theoretical requirement.

A street migrates downstream by the shedding of alternate (Bairstow, 1939, Goldstein, 1938) positive and negative vortices from the two ends of the *equivalent bluff obstacle.* In fact, the passage of a train of alternate cyclones and anticyclones, as obtains in the surface isobars of high latitudes, is strongly reminiscent of an atmospheric street migrating. In the Indian region, however, a chain of migrating " sources " and "sinks" is not in evidence in the surface isobaric charts. Nevertheless, the atmospheric streets aloft, drawn from upper winds, reveal vortices migrating at intervals.

Single Vortex.—A single vortex is shed at the end of a dilatation axis, often accompanied by weather. It is observed that a point vortex shed outside a cyclic field, is transported away in the sinuous streamlines of the directive field.

Vortex Pair.—Two rectilinear vortices of equal strength and opposite signs, when they are close together in a medium of U_0, constitute a *vortex pair.* If one vortex in a pair becomes alternately weaker and stronger than the the other, the system may be called a *degenerate pair.* If the vortices in a pair are small, a doublet may form by their coming close (Ramsay, 1935) together. A pair may appear in the same street as between two converging streets from the same direction, having a phase difference of about 180°. A vortex pair has no cyclic field.

Examples of vortex pairs are common in the transition months, when streets from the same direction usually predominate. For instance, on the morning of the 15th June, 1944, the positive core-cylinder of a vortex pair was over Kathiawar and the negative core-cylinder was over West Rajputana and they extended at least to the 6 km level. Ice accretion was reported between Surat and Jamnagar at about 3-4 km level and the horizontal gradient of potential temperature was feeble around the pair. This picture is consistent with the characteristic streamline pattern associated with a pair. In the daily charts, isobars drawn at 1 mb interval may often reveal a pair. In a medium of irrotational air U_0, the vortices will move in parallel directions. The track of a degenerate pair is sinuous.

Vortex Twin.—It has already been asserted elsewhere that a "locking" or "belting" of opposite streets takes place in a medium of U_0, the neutral air. The belting process is somewhat as follows.

Let two rectilinear vortices of the same sign (say two electric charges of the same sign, as an anology), find themselves in a medium of neutral air. In the case of cyclogenesis, this often occurs in the seasonal trough when U_4 is stationary and in the north U_2 or U_3 is migrating according to season. The charts suggest that the vortex twin in U_0 begins to revolve round a centre on the line joining the two vortices and gives rise to a *belt* in the neutral air. The twin vortices may be considered to be secluded inside the belt. As a matter of fact, two contiguous core-cylinders of the same sign, belonging to two opposite streets and having a phase difference of 180°, belted together, may be called a *twin.* Two cyclonic core-cylinders form a *positive twin* and two anticyclonic ones a *negative twin.*

The Neutral Cylinder.—It should be noted that two (vortex) sources or two sinks, rotating like a "dumb-bell" in U_0 are not strictly equal in the daily charts. In these cases the contour of the belt is nearly oval. In three dimensions, the rotating neutral air belt may be called the *neutral cylinder.* Briefly speaking, the isopleths of wind speed in a cyclic field, drawn in the direction of line of motion in the field, show that the belt is, for all practical purposes, a "a forced vortex" and looks like an "expanded vortex" in the charts. When the point vortices generating the belt are themselves very close together, the belt tends to become circular.

In actual practice, the neutral cylinder is to be regarded as a solid cylinder, the circulation round which is represented by the inner periphery of the *mantle* or the simple vortex. A neutral cylinder and the mantle thus constitute the Rankine " Combined Vortex " (Ramsay, *loc cit.*). The centre of rotation of the dumb-bell is therefore the centre of a solid rotation or the " no wind " centre and the solid rotation portion moves faster than its mantle.

Coloval.—If one of the undisturbed streams diffuses after preliminary belting of a twin (positive or negative) a "disc" (Sen and Ganeshan, 1940) (a neutral cylinder in three dimensions) is left. This is a partly diffused belt. If both the undisturbed streams weaken after belting, a diffused belt or a " coloval " is left. A coloval is thus a diffused solid-rotation.

By the time the neutral air U_0 is destroyed by the diffusion of vorticity, the coloval may be fractured initiating col-activity. In other words, the process simply means that the hitherto secluded rectilinear vortices of the twin may now respond individually to the fluctuations in the respective undisturbed streams. The expression " col-activity " may be further generalised to mean the weather produced by rectilinear vortices of the same sign coming close together in any manner and then retreating from each other.

Cyclone Track in Generative Shell.—The proper perspective of anticyclogenesis and cyclogenesis can be obtained only when the magnitude of free convection along moist adiabates, and free subsidence along dry adiabates (a Carnot's cycle when horizontal motions of air at the surface and aloft are considered to be isothermal), in an extensive generative field is known. Meanwhile, a restricted view regarding the sequence of events in the cyclogenesis field, finally leading to the translation of a cyclone, will be briefly stated.

The sequence of events, as set out below, may differ somewhat in an individual cyclone, according to season. Moreover, the field B, in which the cyclone forms, may quickly come to the convergent state and therefore some of the stages noted below may not be in evidence in a particular case without frequent charts.

Let the westerly undisturbed stream or the monsoon U_4 be diffused in the beginning with U_0 in the seasonal trough or the base a warm anticyclone.

It may be recalled that an important sign of cyclogenesis in the surface isobaric chart is the steepening of the horizontal pressure gradient to the south of the seasonal trough. This corresponds to the diffused westerly street reviving, *i.e.*, the monsoon winds U_4 strengthening, along the inflow axis a_2n_2 in the annexed diagram. The vortex at a_2 finds itself in U_0, which is drawn through convergent channel along n_3a_2.

The easterly street U_3 moves into position. The rectilinear vortices at b_2 and a_2 (a twin) in U_0 form a " dumb-bell " which usually has two unequal heads (vortices of different strengths) and a neutral cylinder is formed.

In the intial stage, the angle $a_2n_2b_2$ is obtuse. The wind gradient, therefore, increases as the neutral point n_2 is approached. Consequently, the neutral cylinder is pushed towards n_2. The portion a_2b_1 of the track (Fig. 1 of the annexed diagram) is traversed.

The "undisturbed stream" of the easterly street (U_3) strengthens. The cyclic field, $n_3a_2n_2b_2$, hitherto diffused, now becomes pronounced and stable with the neutral cylinder inside.

Some vorticity is shed by the *nose* of the coloval (initially pointing towards b_2) into the triangle $b_2n_2a_2$. Strong vortex layers are shed at the ends of the dilatation axis as the mantle begins to grow with the advent of the humidity wave. The wind gradient to the western side of the neutral cylinder thus increases with time.

The angle $b_2n_2a_2$ becomes a right angle and then acute and the wind gradient in the triangle $a_2n_2b_2$ decreases. A thrust on the cylinder is now directed from an easterly to a westerly direction. The portion of the track, t_1A_2 (Fig. 1) is thus traversed at right angles to the axis of U_4. If the angle $b_2n_3a_2$ is obtuse, the cyclone is likely to travel faster than usual.

By the time the neutral cylinder moves some distance towards n_2, determined by wind gradient, the journey is over so far as the travel of a tropical cyclone is concerned. The westward track may be almost as long as the wave length of the participating streets. If conditions continue favourable, a cyclone travelling further westwards should diffuse and then re-intensify in the next cyclic field $n_4a_3n_3b_3$.

Recurving Cyclones.—A cyclone "recurves" when it passes from U_E to U_W of the "general circulation." The herring-bone line in the diagram represents the track of a recurving cyclone. It is observed from the charts that on the eve of recurvature the equatorial U_3 diffuses (a cyclone may initially form between U_3 and U_4 in any season) and the cyclic field $n_2b_2n_3a_2$ opens out to form the *pseudo-cyclic field* $n_7c_2n_3a_2n_2c_1$ so that the cyclone begins to move to the new neutral point n_7. The portion A_2B_2 of the track is thus traversed. Fig. I shows the familiar S-track of cyclones which, as observations show, is independent of the diameter of the neutral cylinder.

Directive Field.—This is the "constant density" layer. The preceding paragraphs explain the trend of motion of a cyclone owing to forces arising in the generative field. The streamlines in directive field above the cyclone past the contour of the core-cylinders, however, indicate the direction of the external force. The streamlines in the directive field are to be regarded as representing the "translation" of a twin in a uniformly rotating fluid in the generative shell.

The annexed diagram depicts the average streamlines for December. The streamlines on individual occasions are similar. It is a curious fact that a cyclonic twin should move anticyclonically in a cyclonic cyclic field, in case the directive drive so dictates. Every streamline in the directive field is a potential cyclone track and available records of tracks of cyclones in the Bay of Bengal and the Arabian Sea and those of the western disturbances in the north are in agreement with this view. As the annexed diagram shows, some of the streamlines are cyclonically while others are anti-cyclonically curved. In actual forecasting, the streamlines at the 6 km. level have been used with satisfactory results.

TRANSLATION OF A WINTER CYCLONE

(DECEMBER)

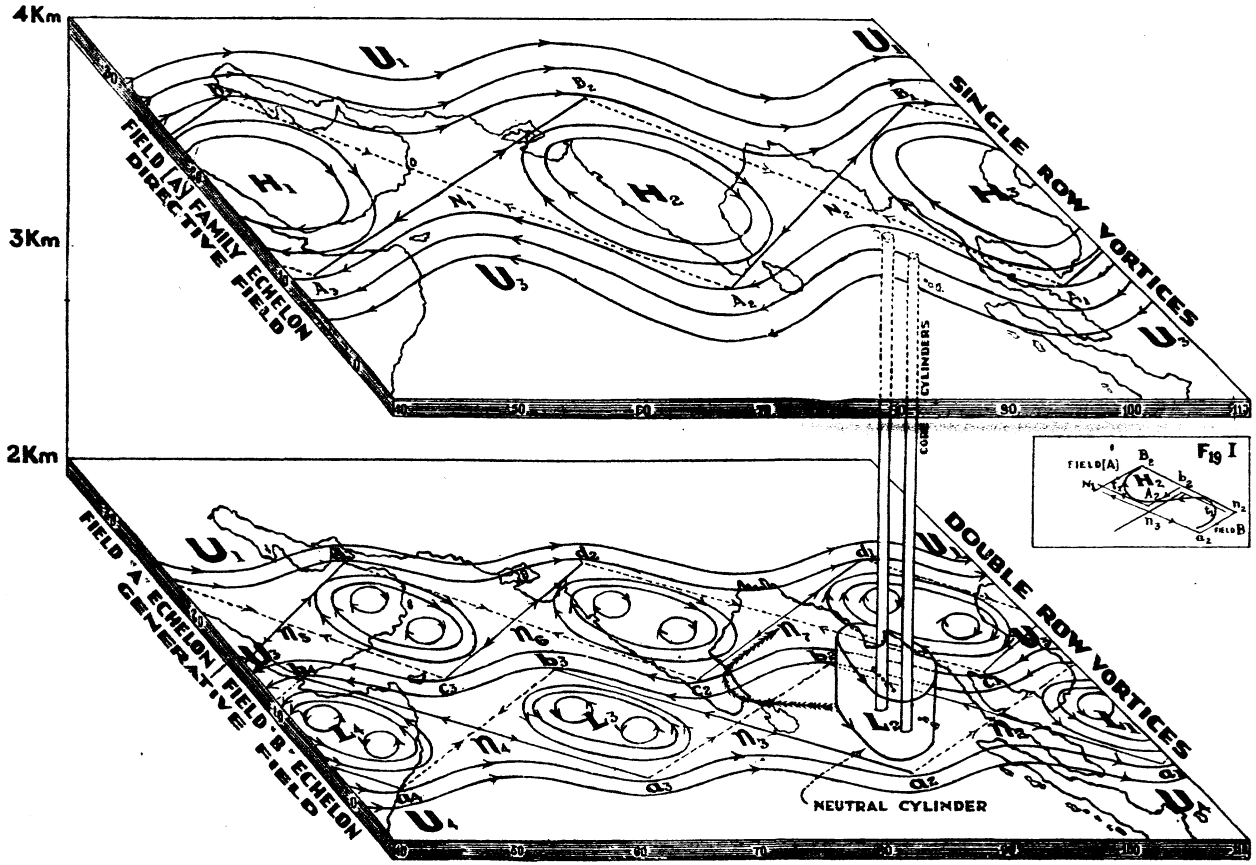

FIG. I

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It may be recalled in this connection that a street of single row vortices (Bairstow, *loc. cit.*) is unstable and this is an important feature of the directive field. As a matter of fact, horizontal gradient of potential temperature aloft are found to be weak in the neighbourhood of the vortices. The twin may change "drive" from cyclonic to anticyclonic and vice versa owing to the streamlines in the directive field becoming unsteady.

If anticyclogenesis develops (*i.e.* spiral descent of air) in the directive shell in one part (*e.g.* H_2 of the directive field) and a cyclone (or spiral ascent of air) is below the field in another part (*e.g.* L_2 of the generative field), the streamlines in the directive field become unsteady. The anticyclonic ceiling of the cyclone in the directive field is thus lowered. The sloping ceiling $N_1B_2N_2A_2$ extend southwards sufficiently to bar the way of the cyclone advancing westwards. The daily charts show that the directive cyclic field $B_2N_2A_2N_1$ or H_2 often extends southwards in the post-monsoon months. It may be recalled that the isentropes appreciably slope down from north to south in this season. The cyclone then halts for a time at the anticyclonic wall (dilatation) and then recurves at comparatively low latitudes (about lat. 15°N on the average in pre-monsoon and post-monsoon months).

It may be observed that the generative and directive fields are "parallel walls" at right angles to the axis of the neutral cylinder. In case the mantle and the neutral cylinder begin to rotate uniformly and sufficiently fast without relative motion, the cyclone would move with the same velocity as obtains in the appropriate streamlines above the two core-cylinders in the directive field. It may be assumed that there is no difference in density between the neutral cylinder and its mantle in any level.

The variation of horizontal velocity of the equatorial along the z axis is small and the average normal velocity in the xy plane of the directive field is roughly of the order of 400 km. per day. A tropical cyclone, therefore, may move as one entity with this average velocity.

Vorticity.—The question of uniform translation of a cyclone (or anticyclone) by the directive field arises only when mass can be "annihilated" in the region of vorticity inside the generative cyclic field so that the mantle may appear mostly outside the cyclic field. The wet phase of the humidity wave fulfills the requirements and its volume and intensity apparently determines the extent of the mantle.

The vorticity accumulates at the four corners of the cyclic field, outside the belt. In the wet phase of a humidity wave, this region of vorticity gives rise to a simple vortex or *mantle* round the belt, which in due course begins to contract. The mantle thus consists of transitional air masses.

In case there is a marked relative motion between the mantle and the neutral cylinder, a vortex layer will arise at the Rankine joint which in daily charts is a region of heavy rainsqualls. This is an important feature of the tropical cyclones.

Humidity Wave.—It is interesting to observe that high pressure centres of the warm anti-cyclonic chain along latitude 15° lie between two air masses, the equatorial U_3 and the tropical U_1. The negative twin in each anticyclone, *e.g.*

H_1 H_2 or H_3 is known to prcduce the strongest "humidity wave" in three dimensions when both the undisturbed streams strengthen. Layers from these sources enter cyclogenesis field.

A brief "life history" of the generative field, cyclonic and anticyclonic, in the shape of changes of surface vapour pressure or minimum temperature (in the absence of rain and katabatic flow) in 120 hrs. usually provides the background of a three dimensional picture of the humidity wave. The "current changes" of these elements in 24 hrs. are helpful in interpreting the "life history."

Vortex pairs simulating cyclones.—A recurving cyclone, as a rule, weakens rapidly. While approaching the dilatation axis of the directive field, one of the core-cylinders may be in a streamline of anticyclonic curvature and the other in a streamline of cyclonic curvature. The vortices in the twin may thus separate from each other. Moreover the dilatation axis of the field [A] (square brackets indicating directive functions) is divergent and of anticyclonic vorticity. A change of drive, therefore, is usually responsible for the weakening of a cyclone. A diffusion of U_3 or U_4 etc. may also contribute to the rapid weakening of a recurving cyclone while passing to U_W. Moreover, a cyclone may also lose its free convection threshold for moist air in U_E while recurving into U_W.

Let a positive twin in the positive cyclic field $n_3a_2n_2b_2$, for example, and a negative twin in the negative cyclic field $n_6d_2n_7c_2$ be simultaneously translated. It may so happen that the positive twin may meet the negative twin on c_2n_7 and pair off, in case the cyclic field is weakening. The two pairs may then travel in opposite directions.

As regards examples of pairing off of twins in the daily charts, it may be recalled that very often weather is unsettled in the north and southwest Bay of Bengal simultaneously, shortly after the passage of a cyclone, giving the wrong impression of a quick and fresh cyclonic formations. Numerous examples of this phenomenon are to be found in the October charts.

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REFERENCES

Bairstow, L., 1939, Applied Aerodynamics, p. 290.

Glauert, H., 1926, Aerjoil and Airscrew Theory, p. 56.

Goldstein, S., 1938, Modern Developments in Fluid Dynamics, p. 557.

Lamb, H., 1930, Hydrodynamics, p 202.

Ramsay, A. S., 1935, A Treatise on Hydromechanics, Pt. II, London, p. 244.

Sen, S. N., 1943, *Science and Culture*, Vol. IX, 90.

Sen, S N., & Ganesan V, 1940, *Science and Culture*, Vol. X, August.